OC Problem Set 5

Changed Tutorial Time: Thursday, May 28, 2015, 9:45am, Kleiner Hörsaal A, Bldg. 30.22

Problem 1: Eigenvalue Equation and Modal Cutoff

The eigenvalue equation for a symmetric slab waveguide is given in the lecture notes by Eq. (2.40) and (2.41), respectively.

a) Write the eigenvalue equation as a function of B and V by using the notation of Eq. (2.38) from the lecture notes (cf. Fig. 2.9b).

b) In the slab waveguide of Problem Set 4 (Problem 1), up to which wavelength λ only the fundamental modes (two polarizations) can propagate ($h = 10\ \mu\text{m}$, $n_1 = 1.45$ and $n_2 = 1.44$)?

c) Now, light with a wavelength of $\lambda = 800\ \text{nm}$ is coupled into the slab waveguide. How many modes can propagate?

Problem 2: Waveguide Dispersion

a) Besides material dispersion, also the mode-dependent waveguiding has to be taken into account to describe chromatic dispersion in a waveguide. This is stated through Eq. (2.54) in the lecture notes.

Derive the expression

$$\frac{t_g}{L} \approx \frac{n_{2g}}{c} + \frac{n_{1g} - n_{2g}}{c} \frac{\mathrm{d}(VB)}{\mathrm{d}V}$$

for the group delay t_g of a mode after propagation through a waveguide of length L

$$\frac{t_g}{L} = \frac{\mathrm{d}\beta}{\mathrm{d}\omega} = \frac{1}{c}\frac{\mathrm{d}\beta}{\mathrm{d}k_0}$$

by using the following approximations:

$$\Delta << 1, \qquad \frac{\mathrm{d}V}{\mathrm{d}k_0} \approx \frac{V}{k_0}, \qquad n_{1g} - n_{2g} \approx n_1 - n_2 .$$

b) Starting from Eq. (2.21) in the script,

$$\frac{t_g}{L} = \frac{\mathrm{d}\beta}{\mathrm{d}\omega} = \frac{1}{c}\frac{\mathrm{d}\beta}{\mathrm{d}k_0},$$

derive the resulting time delay of 2 waves at wavelengths λ_1 and λ_2 in dependence of the material dispersion coefficients of 1st, 2nd, …, nth order. Calculate Δt_g for a propagation length of 15 km for spectral components separated by Δf = 125 GHz at a center wavelength of 1550 nm and a material dispersion coefficient M = 22 ps/(nm km).

c) Explain in words: Intramodal dispersion and intermodal dispersion.

d) Calculate the length of a dispersion-compensating fibre (DCF) with the aim to compensating the accumulated chromatic dispersion after a transmission over L_1 = 250 km through a standard singlemode fibre (SMF). Assume a DCF with a dispersion coefficient $C_{DCF} = -50\,\text{ps/km nm}$.

Problem 3: Pulse Broadening

A light pulse with Gaussian envelope $P_{\text{in}}(t)$ is launched into a medium with baseband impulse response $h(t)$ and a group delay t_g. Determine the output pulse envelope. Show that the output is also a Gaussian and specify the width of the output pulse.

$$P_{\text{in}}(t) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{t^2}{2\sigma_1^2}\right) \quad , \qquad h(t) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{(t-t_g)^2}{2\sigma_2^2}\right)$$

For questions and suggestions on the OC tutorial please contact:

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